

Precalculus

2-01 Complex Numbers

Imaginary Unit

- $i =$ _____
- $i^2 =$ _____

Complex Number

- $a + bi$
- a is _____ part
- bi is _____ part

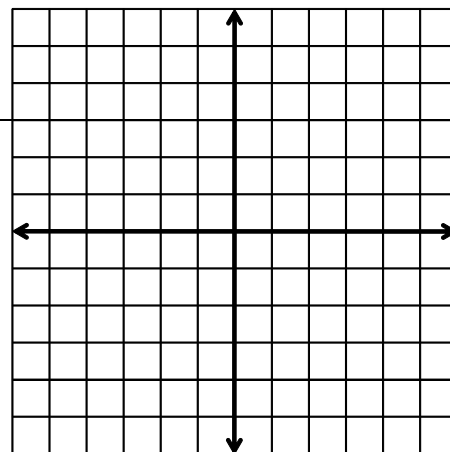
Graphing complex points

- Complex plane
- Horizontal axis = _____
- Vertical axis = _____

Graph

a. $3 + 2i$

b. $-2 - 4i$



Operations

Add/Subtract

- _____ like terms

$$(2 + 3i) + (6 - 2i)$$

Multiplication

- i^2 becomes _____

$$(2 + 3i)(6 - 7i)$$

$$(3 + i)(3 - i)$$

Complex Conjugates

- $a + bi$ and _____
- When complex conjugates are _____, the product is _____

- Multiply by conjugate of denominator

$$\frac{2 - 3i}{4 + 6i}$$

Simplify $(5 - i)^2$

$$\sqrt{-14}\sqrt{-2}$$

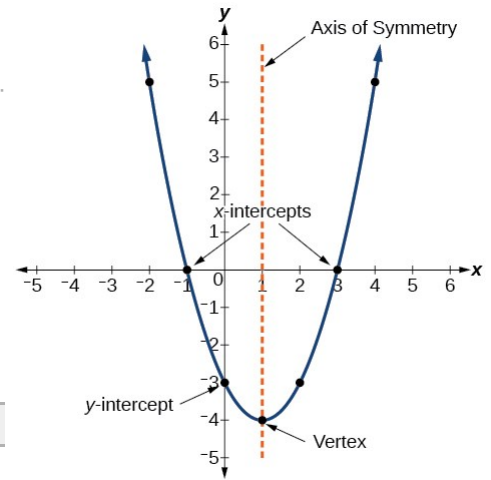
$$\sqrt{-27} - \sqrt{-12}$$

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2-02 Quadratic Equations

$$f(x) = ax^2 + bx + c$$

- $|a| > 1 \rightarrow$ _____
- $0 < |a| < 1 \rightarrow$ _____
- $a < 0 \rightarrow$ _____ over x-axis "opens _____"
- $a > 0 \rightarrow$ "opens _____"



Standard Form

$$f(x) = a(x - h)^2 + k$$

- Vertex _____
- Axis of symmetry _____

To graph

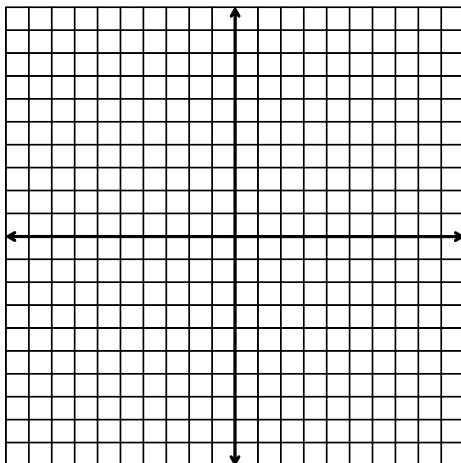
1. Find _____
2. Make _____ around vertex
3. Draw _____

General Form

$$f(x) = ax^2 + bx + c$$

- Vertex _____
- _____
- Axis _____

Graph $f(x) = x^2 - 10x + 25$ and identify the vertex and axis of symmetry



Write the standard form of the equation of parabola with vertex $(-4, 11)$ and passes through $(-6, 15)$

Maximum and minimum

- Occurs at the _____

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve $8x^2 + 14x + 9 = 0$

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2-03 Polynomial Equations





Polynomial Function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

- a_n are _____
- $a_n x^n$ are _____
- a_0 is _____ term
- Degree is _____ exponent
- Leading coefficient is coefficient of term with _____ exponent
- Graphs are _____, _____, _____ turns

End Behavior

- Polynomial functions always go towards _____ or _____ at either _____ of the graph

	Leading Coefficient +	Leading Coefficient -
Even Degree		
Odd Degree		

What is the end behavior of $f(x) = \frac{1}{3}x^3 + 5x$?

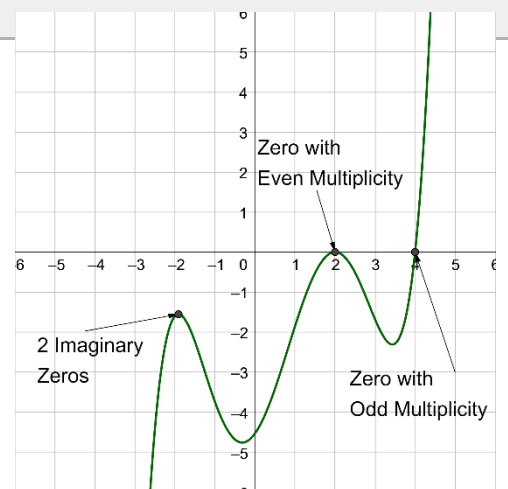
Zeros

If $x = a$ is a zero of $f(x)$

- $x = a$ is a solution to _____
- $(a, 0)$ is _____
- $(x - a)$ is a _____ of $f(x)$

Let $f(x)$ have degree n

- At most n _____ (can be repeated)
- At most $n - 1$ _____ points

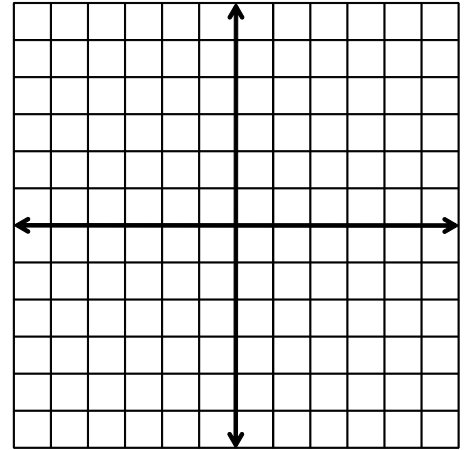


For $g(t) = t^5 - 6t^3 + 9t$

a. Find all zeros

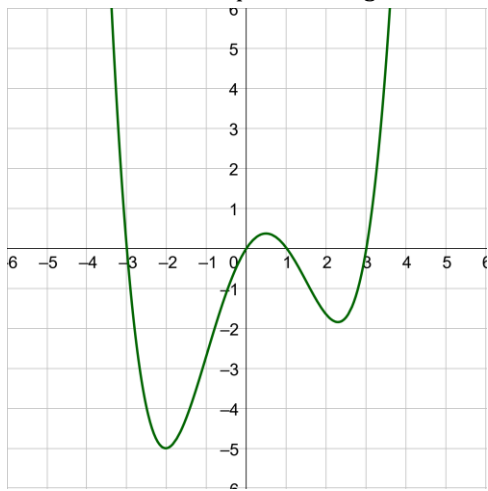
b. Find multiplicity of zeros

c. Graph



Find the intercepts of $f(x) = x(x + 2)(x - 3)$

Determine the least possible degree of the polynomial function shown.



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2-04 Dividing Polynomials

Long Division

Done like long division with _____

$$\frac{y^4 + 2y^2 - y + 5}{y^2 - y + 1}$$

$$\frac{x^3 + 4x^2 - 3x + 10}{x + 2}$$

Synthetic Division

- Shortened form of long division for dividing by a _____
- Only when dividing by _____

$$(-5x^5 - 21x^4 - 3x^3 + 4x^2 + 2x + 2)/(x + 4)$$

$$(y^5 + 32)(y + 2)^{-1}$$

Factor Theorem

If $f(x)$ is divided by $(x - k)$ and remainder is _____, then $(x - k)$ is a _____ of $f(x)$

Show that $(x + 3)$ is a factor of $x^3 - 19x - 30$. Then find the remaining factors.

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2-05 Rational Zeros of Polynomial Functions

Remainder Theorem

If $f(x)$ is divided by $(x - k)$, then the remainder is $r = f(k)$

Use the remainder theorem to evaluate $f(x) = 4x^3 + 10x^2 - 3x - 8$ for $f(-1)$.

Rational Zero Theorem

If polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

The rational zeros are in the form $\frac{p}{q}$

where $p =$ factors of a_0

$q =$ factors of a_n

Find the rational zeros of $f(x) = x^3 - 5x^2 + 2x + 8$ given that $x + 1$ is a factor.

Find the real zeros of $f(x) = x^3 - 7x^2 - 11x + 14$ given that $x + 2$ is a factor.

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2-06 Zeros of Polynomial Functions

Fundamental Theorem of Algebra

If $f(x)$ is polynomial of degree n , then there is at least 1 zero

- There are exactly n zeros
- There are n linear factors (Linear Factorization Theorem)

Find all zeros of $f(x) = x^4 - 16$

Find all the zeros of $f(x) = 2x^4 - 9x^3 - 18x^2 + 71x - 30$

Descartes's Rule of Signs

Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial with real coefficients and $a_0 \neq 0$

The number of _____ real zeros is equal to the number of variations in sign of _____ or less by even integer

The number of _____ real zeros is equal to the number of variations in sign of _____ or less by even integer

Describe the possible real zeros of $f(x) = -2x^3 + 5x^2 - x + 8$

Complex Conjugate Theorem

If a complex number $a + bi$ is a zero, then _____ is also a zero.

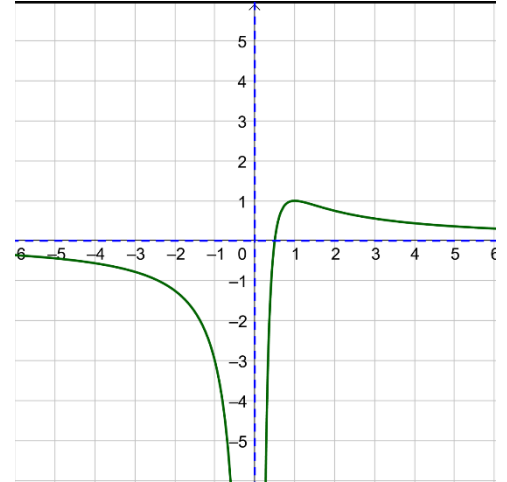
Find a polynomial with real coefficients with zeros $\frac{2}{3}$, -1 , $3 + \sqrt{2}i$

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2-07 Asymptotes of Rational Functions

Rational Function

- _____
- $f(x) = \frac{2x+1}{3x-4}$
- Domain: Denominator _____
- Asymptotes describe behavior of the graph at the _____



Vertical Asymptotes

- _____ and _____
- Set _____ = 0 and solve for x

Horizontal Asymptotes

- Plug in _____ number for x and _____

OR

- Find degree of _____ (N) and _____ (D)
- If $N < D$, _____
- If $N = D$, _____
- If $N > D$, _____

Find the asymptotes of $f(x) = \frac{5x^2}{x^2-1}$

For $f(x) = \frac{2x^2-x}{2x^2+x-1}$

Find the domain

Find the removable discontinuity

Find the asymptotes

Slant Asymptote

- If $N = D + 1$, _____ and _____ remainder

Find the asymptotes of $f(x) = \frac{3x^2+1}{x}$

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2-08 Graphs of Rational Functions

Intercepts

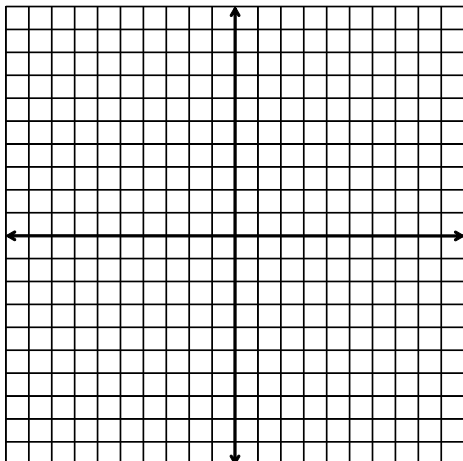
- x-int: let _____
 - Numerator = 0
- y-int: let _____

Find the intercepts of $f(x) = \frac{3x^2-1}{x}$

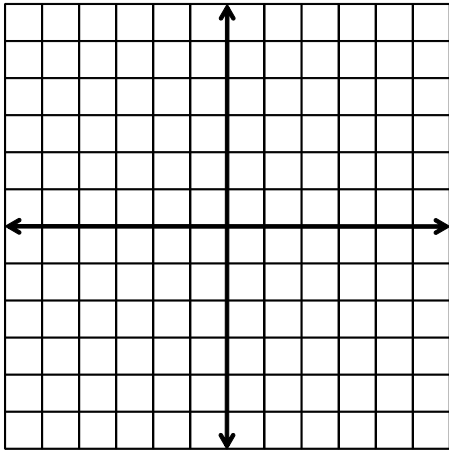
To graph rational functions

1. Find _____
2. Find _____
3. Graph _____ as dotted lines
4. Create _____ of values around asymptotes
5. _____ points
6. Draw curves starting near an _____ and ending near another asymptote
Don't cross _____
7. Put any required _____. Check the _____

Graph $f(x) = \frac{3x^2+1}{x}$



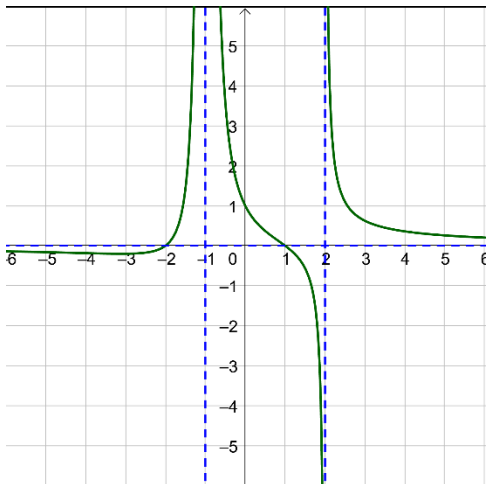
Graph $f(x) = \frac{3x}{x^2+x-2}$



Find the function given a graph

1. Use the x-intercepts and multiplicity to get factors of _____
 - a. If cross x-axis: multiplicity 1 or 3
 - b. If touch but not cross: multiplicity 2 or 4
2. Use vertical asymptotes to get factors of _____
 - a. If 1 end goes up and 1 down: multiplicity 1
 - b. If both ends go same direction: multiplicity 2
3. Use any other point to get _____ factor, a

Find the function



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2-09 Nonlinear Inequalities

Steps to solve

1. Make one side _____
2. Find _____
 - Zeros
 - Undefined
3. Plot on _____
4. Pick _____ points and _____ them

Solve $x^2 - 5x - 6 < 0$

Solve $3x^3 - 4x^2 - 12x > -16$

Solve $\frac{3x-5}{x-3} \leq 1$