Mr. Wright's Math Extravaganza

Precalculus

Polynomial Functions

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on applications

Score	I Can Statements
4.0	☐ I can demonstrate in-depth inferences and applications that go beyond what was taught.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content.
3.0	☐ I can solve polynomial equations. ☐ I can solve nonlinear inequalities.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content.
2.0	☐ I can write square roots of negative numbers as multiples of *i*. ☐ I can do algebraic operations with complex numbers. ☐ I can write and graph quadratic functions. ☐ I can find the minimum or maximum of a quadratic function. ☐ I can write and graph polynomial functions. ☐ I can divide two polynomial expressions. ☐ I can find the rational zeros of a polynomial equation. ☐ I can find the asymptotes of rational functions. ☐ I can graph rational functions.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

Richard Wright

Precalculus

2-01 Complex Numbers

Imaginary Unit

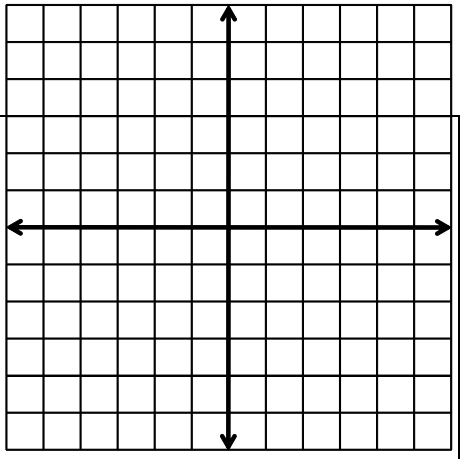
- $i =$_____
- $i^2 =$_____

Complex Number

- $a + bi$
- a is _____ part
- bi is _____ part

Graphing complex points

- Complex plane
- Horizontal axis = _____
- Vertical axis = _____



Graph

a. $3 + 2i$

b. $-2 - 4i$

Operations

Add/Subtract

- _____ like terms

$(2 + 3i) + (6 - 2i)$

Multiplication

- i^2 becomes _____

$(2 + 3i)(6 - 7i)$

$(3 + i)(3 - i)$

Complex Conjugates

- $a + bi$ and _____
- When complex conjugates are _____, the product is _____

Division

- Multiply by conjugate of denominator

$\frac{2-3i}{4+6i}$

Simplify $(5-i)^2$

$\sqrt{-14}\sqrt{-2}$

$\sqrt{-27}-\sqrt{-12}$

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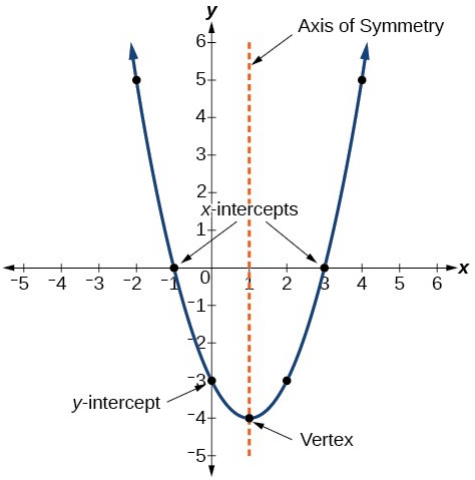
2-02 Quadratic Equations



$$f(x) = ax^2 + bx + c$$

- $|a| > 1$ → _____
- $0 < |a| < 1$ → _____
- $a < 0$ → _____ over x-axis "opens _____"
- $a > 0$ → "opens _____"

Standard Form

$$f(x) = a(x-h)^2 + k$$

- Vertex _____
- Axis of symmetry _____

To graph

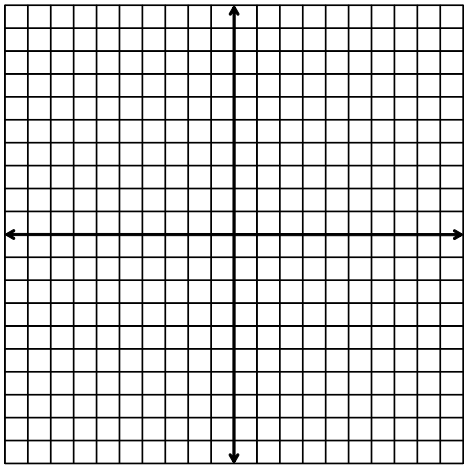
1. Find _____
2. Make _____ around vertex
3. Draw _____

General Form

$$f(x) = ax^2 + bx + c$$

- Vertex _____
-
- Axis _____

Graph $f(x) = x^2 - 10x + 25$ and identify the vertex and axis of symmetry



Write the standard form of the equation of parabola with vertex $(-4,\ 11)$ and passes through $(-6,\ 15)$

Maximum and minimum

- Occurs at the _____

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve $8x^2 + 14x + 9 = 0$

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2-03 Polynomial Equations

Polynomial Function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

- a_n are _____
- $a_n x^n$ are _____
- a_0 is _____ term
- Degree is _____ exponent
- Leading coefficient is coefficient of term with _____ exponent
- Graphs are _____, _____, _____ turns

End Behavior

- Polynomial functions always go towards _____ or _____ at either _____ of the graph

	Leading Coefficient +	Leading Coefficient –
Even Degree		
Odd Degree		

What is the end behavior of $f(x) = \frac{1}{3}x^3 + 5x$?

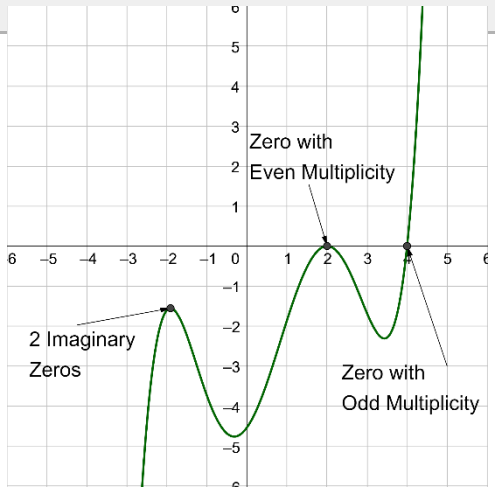
Zeros

If $x = a$ is a zero of $f(x)$

- $x = a$ is a solution to _____
- $(a,\ 0)$ is _____
- $(x - a)$ is a _____ of $f(x)$

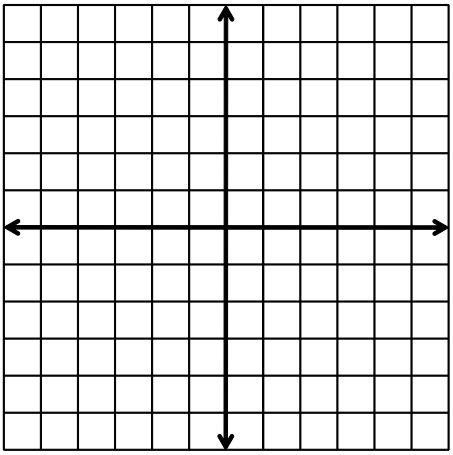
Let $f(x)$ have degree n

- At most n _____ (can be repeated)
- At most $n - 1$ _____ points



For $g(t) = t^5 - 6t^3 + 9t$

a. Find all zeros

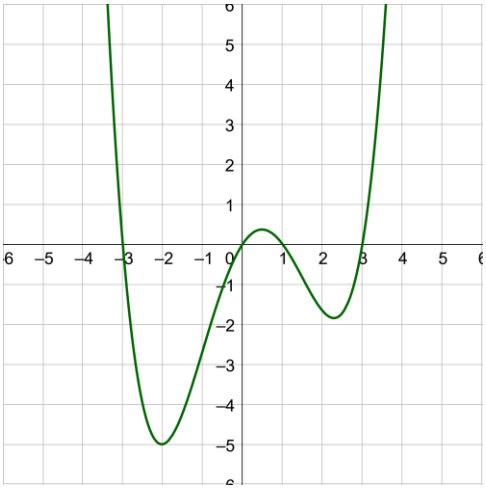


b. Find multiplicity of zeros

c. Graph

Find the intercepts of $f(x) = x(x+2)(x-3)$

Determine the least possible degree of the polynomial function shown.



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2-04 Dividing Polynomials

Long Division

Done like long division with _____

$\frac{y^4+2y^2-y+5}{y^2-y+1}$

$\frac{x^3+4x^2-3x+10}{x+2}$

Synthetic Division

- Shortened form of long division for dividing by a _____
- Only when dividing by _____

$(-5x^5 - 21x^4 - 3x^3 + 4x^2 + 2x + 2)/(x + 4)$

$(y^5 + 32)(y + 2)^{-1}$

Factor Theorem

If $f(x)$ is divided by $(x - k)$ and remainder is _____, then $(x - k)$ is a _____of $f(x)$

Show that $(x + 3)$ is a factor of $x^3 - 19x - 30$. Then find the remaining factors.

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2-05 Rational Zeros of Polynomial Functions

Remainder Theorem

If *f*(*x*) is divided by $(x-k)$**, then the remainder is** $r=f(k)$

Use the remainder theorem to evaluate $f(x)=4x^3+10x^2-3x-8$ for $f(-1)$.

Rational Zero Theorem

If polynomial

$$f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_2x^2+a_1x+a_0$$

The rational zeros are in the form $\frac{p}{q}$

where *p* = factors of a_0

q **= factors of** a_n

Find the rational zeros of $f(x)=x^3-5x^2+2x+8$ given that $x+1$ is a factor.

Find the real zeros of $f(x)=x^3-7x^2-11x+14$ given that $x+2$ is a factor.

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2-06 Zeros of Polynomial Functions

Fundamental Theorem of Algebra

If $f(x)$ is polynomial of degree n, then there is at least 1 zero

- **There are exactly n zeros**
- **There are n linear factors (Linear Factorization Theorem)**

Find all zeros of $f(x) = x^4 - 16$

Find all the zeros of $f(x) = 2x^4 - 9x^3 - 18x^2 + 71x - 30$

Descartes's Rule of Signs

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial with real coefficients and $a_0 \neq 0$
The number of _____real zeros is equal to the number of variations in sign of _____ or less by even integer
The number of _____real zeros is equal to the number of variations in sign of _____ or less by even integer

Describe the possible real zeros of $f(x) = -2x^3 + 5x^2 - x + 8$

Complex Conjugate Theorem

If a complex number $a + bi$ is a zero, then _____ is also a zero.

Find a polynomial with real coefficients with zeros $\frac{2}{3},\ -1,\ 3 + \sqrt{2}i$

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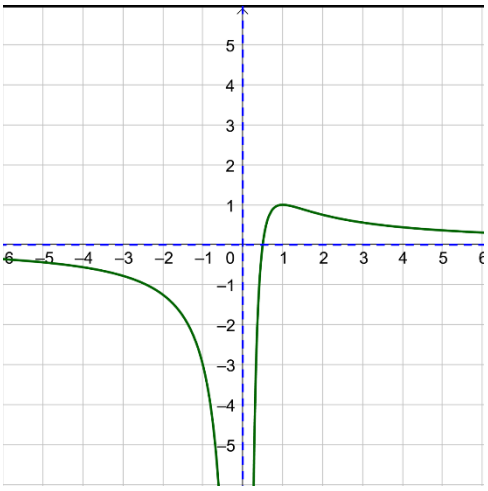
2-07 Asymptotes of Rational Functions

Rational Function

- _____
- $f(x) = \frac{2x+1}{3x-4}$
- Domain: Denominator _____
- Asymptotes describe behavior of the graph at the _____



Vertical Asymptotes

- _____and _____
- Set _____= 0 and solve for *x*

Horizontal Asymptotes

- Plug in _____number for *x* and _____

OR

- Find degree of _____(N) and _____(D)
- If N < D, _____
- If N = D, _____
- If N > D, _____

Find the asymptotes of $f(x) = \frac{5x^2}{x^2-1}$

For $f(x) = \frac{2x^2-x}{2x^2+x-1}$

Find the domain

Find the removable discontinuity

Find the asymptotes

Slant Asymptote

- If $N = D + 1$, _____and _____remainder

Find the asymptotes of $f(x) = \frac{3x^2+1}{x}$

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2-08 Graphs of Rational Functions

Intercepts

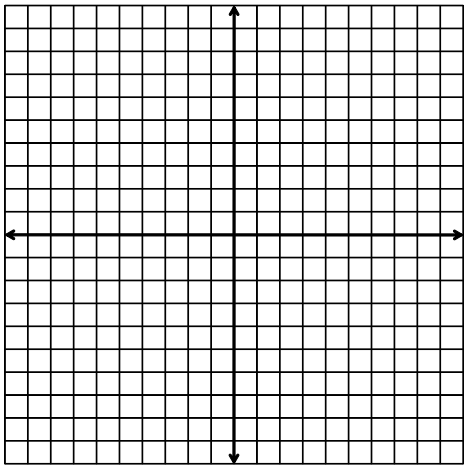
- *x*-int: let _____
  - Numerator = 0
- y-int: let _____

Find the intercepts of $f(x) = \frac{3x^2-1}{x}$

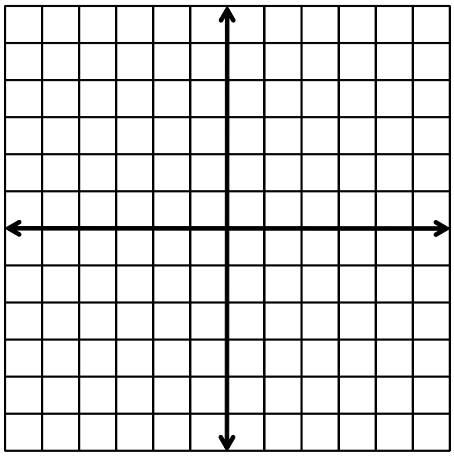
To graph rational functions

1. Find _____
2. Find _____
3. Graph _____as dotted lines
4. Create _____of values around asymptotes
5. _____points
6. Draw cures starting near an _____and ending near another asymptote
   Don't cross _____
7. Put any required _____. Check the _____

Graph $f(x) = \frac{3x^2+1}{x}$



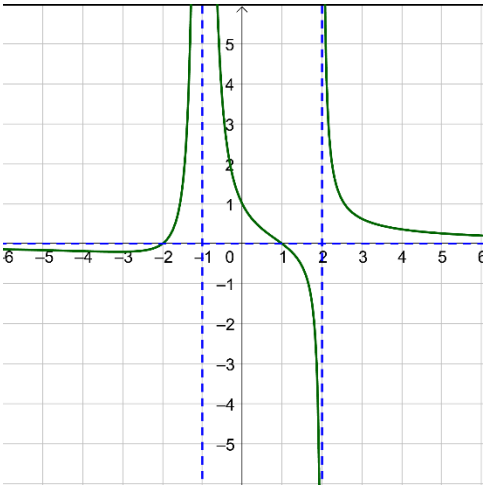
Graph $f(x) = \frac{3x}{x^2+x-2}$



Find the function given a graph

1. Use the x-intercepts and multiplicity to get factors of _____
   a. If cross x-axis: multiplicity 1 or 3
   b. If touch but not cross: multiplicity 2 or 4
2. Use vertical asymptotes to get factors of _____
   a. If 1 end goes up and 1 down: multiplicity 1
   b. If both ends go same direction: multiplicity 2
3. Use any other point to get _____factor, *a*

Find the function



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2-09 Nonlinear Inequalities

Steps to solve

1. Make one side _____
2. Find _____
   - Zeros
   - Undefined
3. Plot on _____
4. Pick _____points and _____them

Solve $x^2 - 5x - 6 < 0$

Solve $3x^3 - 4x^2 - 12x > -16$

Solve $\frac{3x-5}{x-3} \leq 1$